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Dynamic Load Carrying Capacity of Flexible Joint Robot

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Abstract

A computational technique for obtaining maximum load carrying capacity of robotic manipulators with joint elasticity subject to accuracy and actuators constraints is described, while Feedback Linearization technique is used to minimize end-effector deflection. An inversion algorithm is employed for the synthesis of a dynamic feedback control law that gives input-output decoupling and full state linearization. The linearizing input transformation and corresponding state diffeomorphisms are presented. Then, this technique is applied for a class of flexible joint robots. In some case studies, linearizing control law is expressed in terms of different set of model variables and their derivatives. As a result, different tracking errors and different torques are introduced in the robot given trajectory, and so different load carrying capacity is obtained in each case.

Keywords: Feedback Linearization; Flexible joint; Elastic; Dynamic Load; Robot

1. Introduction

It is well known that the main source of vibration in industrial robot manipulators is the presence of joint elasticity between the driving actuators and the driven links[1, 2]. The main aims of now a day's robot controllers are accurate and stable tracking of a given desired trajectory regardless of structural flexibility. These types of controllers should be designed on the basis of a more complete dynamic model of the robot [3]. The modeling of robots with elastic joints dates back to the early 80's [4]. More recently, a detailed analysis of the model structure has been given in [5], where it was used for proving asymptotic stability of a simple regulation controller. Also, a reduced dynamic model has been introduced in [6], ignoring gyroscopic terms in the kinetic energy of the motors.For many industrial applications, current robotic manipulators with joint elasticity are relatively slow even when they are not fully loaded. Their speed, load carrying capacity and their productivity are limited by the deflection of the end-effector and the capability of their actuators. Increasing actuator size and power is not the answer, and it would be largely self-defeating. This is due to the increased cost and power consumption of the larger actuators and also due to the increased inertia of the actuators themselves. Thomas et al. [7] have used the load capacity as a criterion for sizing the actuators at the design stage for robot manipulators. In this work, piecewise rigid links and joints were assumed. If one removes the rigid body assumption, the Dynamic Load Carrying Capacity (DLCC) determined under the actuator constraint alone [8] would normally be too large. In [9] a new method of determining DLCC for flexible joint manipulators subject to both actuator and end-effector deflection constraints introduced.

During the last decades, nonlinear systems and control theory have witnessed tremendous development. As one of the most active research areas, *feedback linearization* is a powerful tool for control and synthesis of nonlinear systems, and has been widely applied to many engineering systems. For example, rigid and flexible joint robots are discussed in [10-11]. Feedback linearization involves transforming a nonlinear system into a controllable linear one, by using state feedback and coordinate transformations. This problem has been studied using more general feedback transformations. Static state feedback linearization was solved in [12] for single input systems and in [13] for multi-input systems. For high order systems, transformation to a state-space form may obscure some relevant model structural properties and lead to complicated expressions [14].

In this paper, a new method of feedback linearization technique is presented for flexible joint manipulators instead of feed forward method. Then, the presented method is used for determining dynamic load carrying capacity of these types of manipulators. In the three different case studies, with considering different links or rotor angles as state feedback variables, linearizing control law are applied. In each case, different tracking error and control signal (torque) is resulted. Consequently, different load carrying capacity is obtained. It is shown that using the feed back linearization technique, the end effector deflection is reduced, and therefore DLCC for a given trajectory is improved.

2. Feedback Linearization Controller

Each multi-input nonlinear system can be written as bellow: [15]

$$\dot{x} = f(x) + \sum_{i=1}^{m} u_i g_i(x) = f(x) + g(x)u \tag{1}$$

Where f(x) and g(x) are analytic functions on R^n , f(0)=0, and $u \in R$. This system is said to be feedback linearizable if there exists a region U in R^n containing the origin, a diffeomorphism $T: U \to R^n$, and nonlinear feedback:

$$u = \alpha(x) + \beta(x)v \tag{2}$$

With $\beta(x) \neq 0$ on U such that the transformed variables:

$$Y=T(x) \tag{3}$$

Satisfy the system of equations:

$$\dot{y}(x) = Ay(x) + bv$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ . & . & . & & . \\ . & . & . & & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \& \; b = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ 1 \end{bmatrix} \tag{4}$$

In fact, the whole feedback linearization problem is determining u, such that system can be linearized. To calculate u, each diffeomorphism transformation would be derivated until u appears. Then, the last derivation must be set equal to v and accordingly calculate u out of it. The value of v can be obtained as below: [15]

$$v = y_d^{(r)} - k_{r-1}(y^{(r-1)} - y_d^{(r-1)}) - k_{r-2}(y^{(r-2)} - y_d^{(r-2)}) - \dots - k_0(y - y_d) \tag{5}$$

Applying this control law to the r[th] order linear system, tracking error $e = y - y_d$ satisfies the r[th] order linear equation. For tracking desired path we must set $v = y_d$. Therefore:

$$e^{(r)} + k_{r-1}e^{(r-1)} + k_{r-2}e^{(r-2)} + \dots + k_0 e = 0 \tag{6}$$

Hence, the error dynamics are completely determined by the choice of gains k. It means with choosing suitable k_i, and *i=1, 2,…, r* tracking error can be minimized.

3. Algorithm of Determining Feedback Linearization

State variables are set as link angles, rotor angles and their derivatives, and identified as $x_1, x_2, \dots, x_m$. As we know, such a system will be shown with first order derivatives of state variables. Thus, the system is of the form (1) and is linearizable. Then with having necessary and sufficient derivation from both part of equation (3) we can obtain:

$$\begin{aligned} \dot{y}_i &= \dot{T}_i = \frac{\partial T_i}{\partial x_1}\dot{x}_1 + \dots + \frac{\partial T}{\partial x_n}\dot{x}_n = C_1(x_1, x_2, \dots x_m) \\ \ddot{y}_i &= \ddot{T}_i = C_2(x_1, x_2, \dots x_m) \\ y_i^{(3)} &= T_i^{(3)} = C_3(x_1, x_2, \dots x_m) \\ &. \\ &. \\ y_i^{(n)} &= T_i^{(n)} = C_n(x_1, x_2, \dots x_m) + D_n(u_1, \dots u_i) = v_i \end{aligned} \tag{7}$$

Where, y_i are the components of Y and T_i are the components of T. The control signal u_i can be obtained from the last derivation of equation (7) as follow:

$$u_i = \alpha_i(x_1, x_2, ..., x_m) + \beta_i(x_1, x_2, ..., x_m, v_1, ...v_i) \quad (8)$$

So, feedback linearization parameters will be obtained and the system can be linearized as shown in Fig. 1.

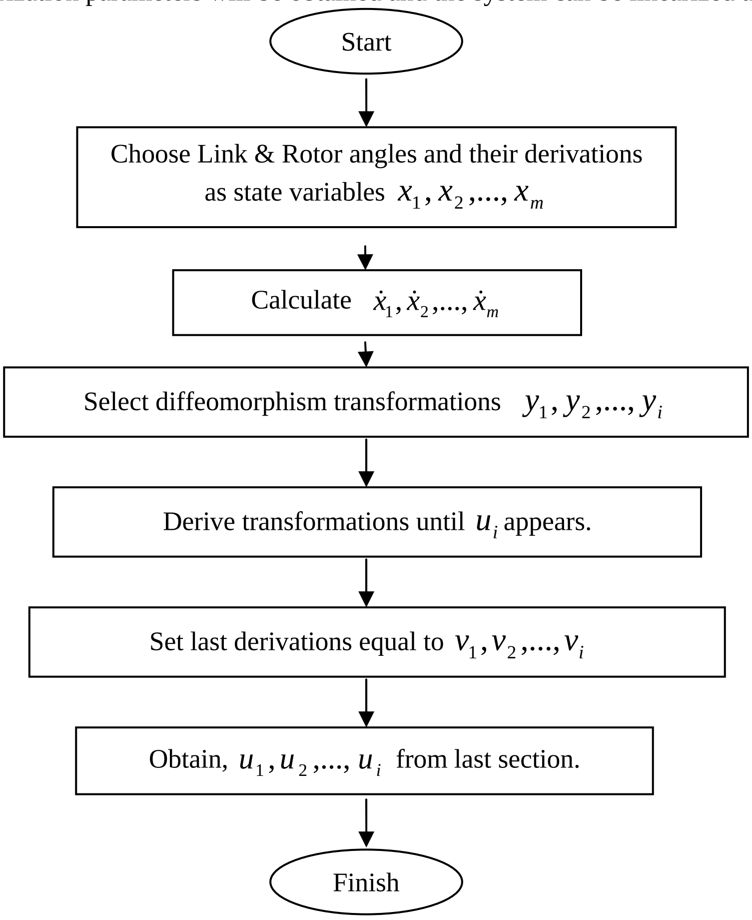

Figure 1 Algorithm for determining feedback linearization.

4. DLCC for a Desired trajectory

For a prescribed trajectory, the DLCC of a flexible joint manipulator is defined as the maximum load that the manipulator can carry in executing the trajectory with an acceptable tracking accuracy. The main constraints, which bound the DLCC of manipulators, are actuators and accuracy constraints.

4.1. Actuator constraint

Actuator constraint based on the typical torque-speed characteristics of DC motors is as follows [9], while other actuation systems characteristics can be applied similarly:

$$\begin{aligned} u_a^+ &= k_1 - k_2\dot{q} \\ u_a^- &= -k_1 - k_2\dot{q} \end{aligned} \quad (9)$$

Where $k_1 = \tau_s, k_2 = \tau_s / w_0$, τ_s is the stall torque, w_o is maximum no load speed of the motor. Now the upper and lower bounds on torques available for load can be expressed as:

$$\begin{aligned} \tau_i^+ &= (u_a^+)_i - (\tau_e)_i \\ \tau_i^- &= (u_a^-)_i - (\tau_e)_i \end{aligned} \quad (10)$$

Then, the maximum allowable torque at joint *i* is equal to:

$$(\tau_a)_i = \max\{\tau_i^+, \tau_i^-\} \tag{11}$$

A load coefficient, complying with the torque constraint can be obtained as follows:

$$(c_a)_j = \min\left\{\frac{(\tau_a)_i}{\max\{\tau_e) - \max\{\tau_n\}}, i = 1,\ldots,n\right\} \tag{12}$$

Where τ_n is the no-load torque.

4.2. End-effector accuracy constraint

At first, the given trajectory is discretized into *m* points, and no load deflection $(\Delta_n)_j$ and deflection with added end effector mass $(\Delta_e)_j$, are calculated for *j=1, 2, ..., m.* Also, R_p indicate how much load can be carried without violating the deflection constraint through point j. Then, the load coefficient $(c_p)_j$ for point j is defined as follows:

$$(c_p)_j = \frac{R_p - (\Delta_e)_j}{\max\{\Delta_e\} - \max\{\Delta_n\}} \tag{13}$$

A load coefficient *c* can be found as follows:

$$c = \min\{(c_p)_j, (c_a)_j\}, \qquad j = 1,\ldots,m \tag{14}$$

So, the maximum mass (m_{load}) for this trajectory is:

$$m_{load} = cm_e \tag{15}$$

Where, m_e is the mass of the end-effector.

4.3. Algorithm of determining DLCC with Feedback Linearization

Algorithm of determining DLCC with Feedback Linearization has two parts. In the first part, the effects of combined manipulator and load motions on actuator torques τ are computed. In the next part, supposing motion of robot along the desired trajectory, corresponding no-load torque τ_{nl} is computed. Subtracting τ_{nl} from τ we can obtain torque needed for carrying the load τ_l as, $\tau_l = \tau - \tau_{nl}$. At first, for determining DLCC value, the desired path is discreted to some points. For these m points, robot kinematical variables are computed. In the next step, the executed kiematic parameters must be checked with their acceptable bound.

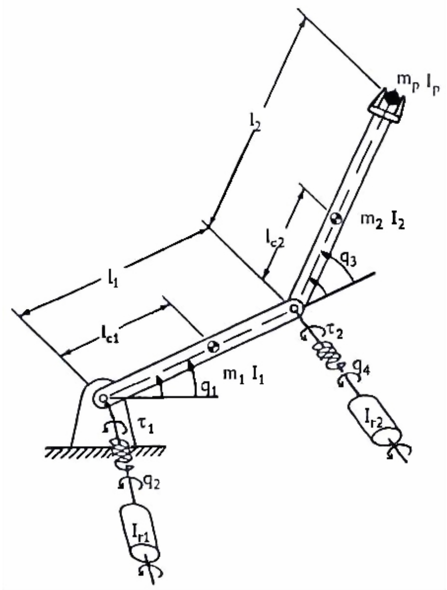

Figure 2 Two-Link flexible joint manipulator model.

5. Controller Design

Equations of a two-link flexible joint manipulator are as below (Fig. 2):

a. Kinematic equations:

$$\begin{aligned}(-l_1 Sin(q_1) - l_2 Sin(q_1+q_3))\ddot{q}_1 - l_2 Sin(q_1+q_3)\ddot{q}_2 &= R_{t1}\\ (-l_1 Cos(q_1) + l_2 Cos(q_1+q_3))\ddot{q}_1 + l_2 Cos(q_1+q_3)\ddot{q}_2 &= R_{t2}\end{aligned} \tag{16}$$

b. Dynamic equations based on Lagrangian approach:

$$\begin{aligned}&D_{11}\ddot{q}_1 + D_{12}\ddot{q}_3 - C\dot{q}_3^2 - 2C\dot{q}_1\dot{q}_3 + K(q_1-q_2) = 0\\ &D_{22}\ddot{q}_3 + D_{12}\ddot{q}_1 + C\dot{q}_1^2 + K(q_3-q_4) = 0\\ &I_{r1}\ddot{q}_2 + K(q_2-q_1) = \tau_1\\ &I_{r2}\ddot{q}_4 + K(q_4-q_3) = \tau_2\end{aligned} \tag{17}$$

Where D_{11}, D_{22}, D_{12}, and C are given in [9]. In state space we set:

$$\begin{aligned}X_1 &= q_1 & X_2 &= \dot{q}_1\\ X_3 &= q_3 & X_4 &= \dot{q}_3\\ X_5 &= q_2 & X_6 &= \dot{q}_2\\ X_7 &= q_4 & X_8 &= \dot{q}_4\end{aligned} \tag{18}$$

Where, q_1, q_3 are link angles and q_2, q_4 are rotor angles. At this time, two-link flexible joint manipulator system (16) is rearranged in state space form as bellow:

$$\begin{aligned}&\dot{X}_1 = X_2;\ \dot{X}_2 = \frac{D_{22}(CX_4(2X_2+X_4)+K(X_5-X_1))+D_{12}(CX_2^2+K(X_3-X_7))}{D_{11}D_{22}-D_{12}^2}\\ &\dot{X}_3 = X_4;\ \dot{X}_4 = \frac{D_{12}(CX_4(2X_2+X_4)+K(X_5-X_1))+D_{11}(CX_2^2+K(X_3-X_7))}{-D_{11}D_{22}+D_{12}^2}\\ &\dot{X}_5 = X_6;\ \dot{X}_6 = \frac{1}{I}(u_1+K(X_1-X_5))\\ &\dot{X}_7 = X_8;\ \dot{X}_8 = \frac{1}{I}(u_2+K(X_3-X_7))\end{aligned} \tag{19}$$

So, the (16) system leads to:

$$\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2, \quad x \in R^n$$

$$f(x) = \begin{bmatrix} X_2\\ \dfrac{D_{22}(CX_4(2X_2+X_4)+K(X_5-X_1))+D_{12}(CX_2^2+K(X_3-X_7))}{D_{11}D_{22}-D_{12}^2}\\ X_4\\ \dfrac{D_{12}(CX_4(2X_2+X_4)+K(X_5-X_1))+D_{11}(CX_2^2+K(X_3-X_7))}{-D_{11}D_{22}+D_{12}^2}\\ X_6\\ \dfrac{K}{I}(X_1-X_5)\\ X_8\\ \dfrac{K}{I}(X_3-X_7)\end{bmatrix} g_1(x) = \begin{bmatrix}0\\0\\0\\0\\0\\ \frac{1}{I}\\0\\0\end{bmatrix}, g_2(x) = \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\ \frac{1}{I}\end{bmatrix} \tag{20}$$

Now, we have different choices for solving the problem of feedback linearization. Because, a few function can be chosen for diffeomorphism transformations. We introduce two methods; in the first one the link angles are selected and in the second one the rotor angles are selected as diffeomorphism transformation.

Method 1- In first method, the diffeomorphism transformations for two-link robot arm are $T_1 = X_1 = q_1$, $T_2 = X_3 = q_3$. Also, Feedback Linearization parameters u_1, u_2 will be obtained with four time deriving of desire values (q_1, q_3). As a result, we have:

$$\begin{aligned} u_1 &= tm_{30}(-v_1 + tm_{23}) + tm_{31}(-v_2 + tm_{25}) \\ u_2 &= tm_{32}(-v_1 + tm_{23}) + tm_{33}(-v_2 + tm_{25}) \end{aligned} \tag{21}$$

Where the relative terms of these equations can be found.

Method 2- In second method, the diffeomorphism transformations for two-link robot arm are $T_1 = X_5 = q_2$, $T_2 = X_7 = q_4$. It means that desired values are (q_2, q_4) and Feedback Linearization parameters are u_1, u_2. Calculation resulted from two times derivation of T_1 and T_2 are as below:

$$\begin{aligned} u_1 &= I.v_1 - K(X_1 - X_5) \\ u_2 &= I.v_2 - K(X_3 - X_7) \end{aligned} \tag{22}$$

6. Simulation

Determining tracking error is possible when we calculate feedback linearization parameters. Thus, we have to gain v_1, v_2 as follow:

$$\begin{aligned} v_1 &= \ddot{q}_{d2} - k_{11}(\dot{q}_2 - \dot{q}_{d2}) - k_{12}(q_2 - q_{d2}) \\ v_2 &= \ddot{q}_{d4} - k_{21}(\dot{q}_4 - \dot{q}_{d4}) - k_{22}(q_4 - q_{d4}) \end{aligned} \tag{23}$$

Now, error diminishes to zero when, $v_1 = \ddot{q}_2, v_2 = \ddot{q}_4$. Hence, error equations are being as bellow:

$$\begin{aligned} \ddot{e}_1 + k_{11}\dot{e}_1 + k_{12}e_1 = 0 \\ \ddot{e}_2 + k_{21}\dot{e}_2 + k_{22}e_2 = 0 \end{aligned} \tag{24}$$

By appropriate selection of $k_{22}, k_{21}, k_{12}, k_{11}$, tracking error can be minimized. We designed close loop system controller to have duplicate poles in -10. It means that we have, $k_{22} = k_{21} = k_{12} = k_{11} = 20$, for simplifying the simulation, we set $I_p = 0$. The desired trajectory selected for simulation as a function of time is as; $X_d = 1.05 - 0.02t^2$, $Y_d = 1.05 + 0.01t^2$. In Figure 3 we have the first controller that tracks link angles and Figure 4 shows desired and actual path with corresponding employed controller. Figure 5 shows the torque of both first and second joint actuators in full load and no load status in the first controller. We have controllers 2 and 3 that tracks rotor angles, where desired and actual path associated to these controller can be shown.

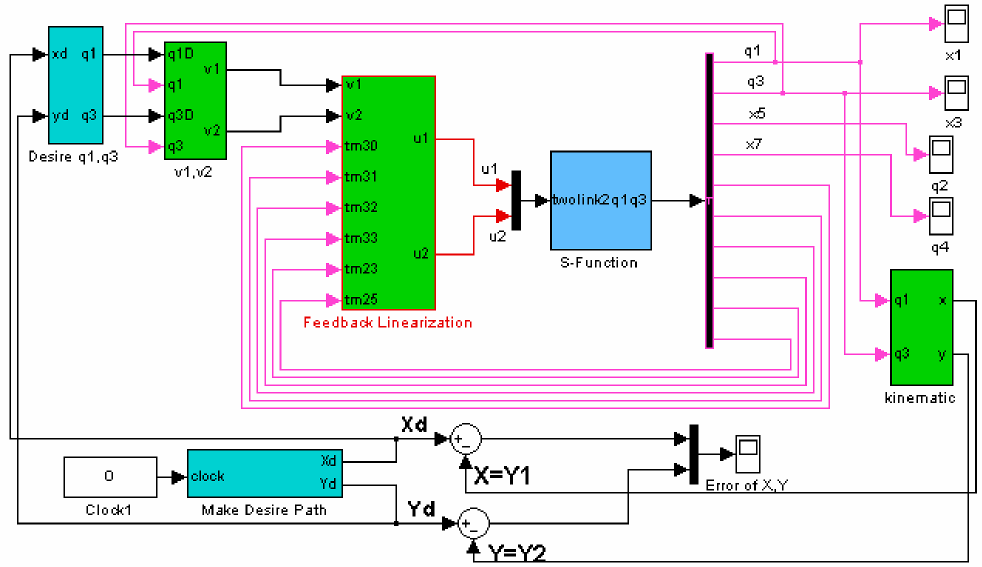

Figure 3 Controller 1which tracks link angles (q1, q3)

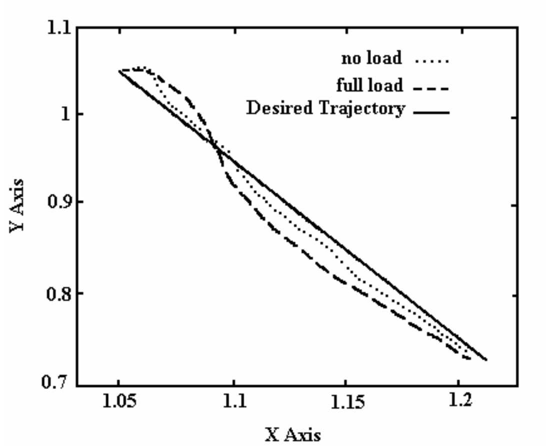

Figure 4 Desired and actual trajectory corresponding to controller 1

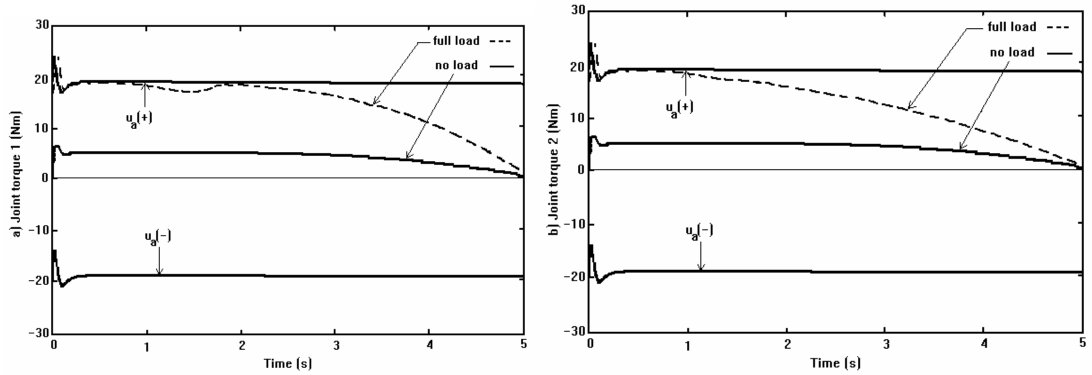

Figure 5 First and Second Joints torques associated to controller 1

Employing the controller 1, the DLCC of manipulator is computed equal to $m_{load} = 1.73$ kg. Applying heavier loads may result tracking error increased to unacceptable condition.

Employing the controller 2, the DLCC of manipulator is computed equal to $m_{load} = 1.7$ kg.

In the last method of simulation, we use (q_1, q_3) separately for tracking trajectory. This can reduce state space variables to four, and hence computational speed will grow up. However, the error is more than before cases. Employing controller 3, DLCC of manipulator is computed as $m_{load} = 1.57$ kg.

6.1. Two-link flexible joint manipulator simulation: Open loop case

In the open loop systems the deflection of joints are more than closed loop systems, which leads manipulator into a smaller DLCC. In this method, the DLCC is found $m_{load} = 0.67$ kg. The path of the robot in this method is shown in Figure 6.

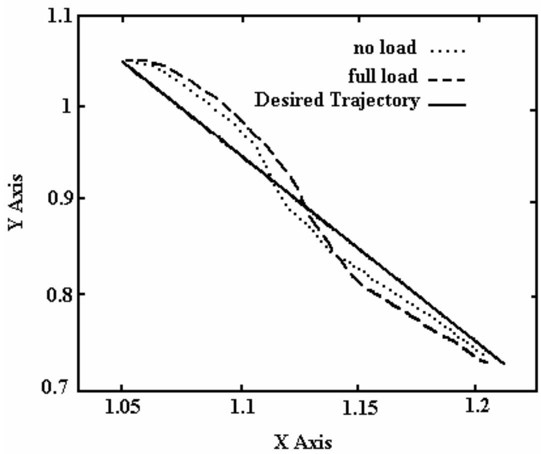

Figure 6 The desired and actual trajectory without Feedback Linearization.

Clearly, we can see that the DLCC for robot was increased by using feedback linearization, and it is because of reduced error and deflection in robot. It is seen, in feedback linearization method, tracking error is minimized in each point of the given trajectory with respect to open loop method. As a result, the dynamic load carrying capacity of manipulator will be increased. In rigid manipulators both inverse dynamics control [15] and the feedback Linearizing control result the same value for dynamic load carrying capacity. In the simulation studies three methods for two-link manipulator with feedback linearization is used, that their differences was in tracking parameters, which produce different tracking errors and control signals (torque of joints). Hence, the maximum load carrying capacity in each case are different. It is seen that with increasing tracking error, we can increase the maximum load carrying capacity of robot.

7. Conclusions

For a class of robots with elastic joints, a new method of feedback linearization is introduced. Three methods have been introduced to simulate two-link robot arm with flexible joint in Feedback Linearization. In each method the DLCC of manipulators has been determined. The use of feedback linearization instead of feedforward method can increase maximum load carrying capacity of robots. It's so simple to increase DLCC, to do this it's needed to reduce deflection. Feedback linearization is such a method that can reduce deflection.
Also, it's shown that replacing poles placement of closed loop system tracking error can be reduced and because of that DLCC will be decreased. So, for each application we can have either of more loads or less error.

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